

Find the solution to $a_n = 7a_{n-2} + 6a_{n-3}$ with $a_0 = 9$, $a_1 = 10$, and $a_2 = 32$.

What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots $1, 1, 1, 1, -2, -2, -2, 3, 3, -4$?

Theorem 4: Let c_1, c_2, \dots, c_k be real numbers. Suppose that the characteristic equation $r^k - c_1r^{k-1} - c_2r^{k-2} - \dots - c_k = 0$ has t **distinct roots** r_1, r_2, \dots, r_t ($t \leq k$) **with multiplicities** m_1, m_2, \dots, m_t , respectively, so that $m_i \geq 1$ for $i = 1, 2, \dots, t$ and $m_1 + m_2 + \dots + m_t = k$. Then a sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \dots + c_ka_{n-k}$ if and only if $a_n = (\alpha_{1,0} + \alpha_{1,1}n + \dots + \alpha_{1,m_1-1}n^{m_1-1})r_1^n + (\alpha_{2,0} + \alpha_{2,1}n + \dots + \alpha_{2,m_2-1}n^{m_2-1})r_2^n + \dots + (\alpha_{t,0} + \alpha_{t,1}n + \dots + \alpha_{t,m_t-1}n^{m_t-1})r_t^n$ for $n = 0, 1, 2, \dots$, where $\alpha_{i,j}$ are constants for $1 \leq i \leq t$ and $0 \leq j \leq m_i - 1$.

Note: Theorems 1, 2 and 3 are corollaries to Theorem 4

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

A) Show that $a_n = -2^{n+1}$ is a solution of this recurrence relation.

B) Use Theorem 5 to find all solutions of this recurrence relation

Theorem 5: If $\{a_n^{(p)}\}$ is a particular solution of the non-homogeneous linear recurrence relation with constant coefficients $a_n = c_1a_{n-1} + c_2a_{n-2} + \dots + c_ka_{n-k} + F(n)$, then every solution is of the form $\{a_n^{(p)} + a_n^{(h)}\}$, where $\{a_n^{(h)}\}$ is a solution of the associated linear homogeneous recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \dots + c_ka_{n-k}$.

What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if **$F(n) = n^3$**

Theorem 6: If $\{a_n\}$ satisfies the linear non-homogeneous recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2} + \dots + c_ka_{n-k} + F(n)$, where $F(n) = (b_t n^t + b_{t-1} n^{t-1} + \dots + b_1 n + b_0) s^n$. When s is **not a root** of the characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form $(p_t n^t + p_{t-1} n^{t-1} + \dots + p_1 n + p_0) s^n$. When s is **a root** of this characteristic equation of **multiplicity** m , there is a particular solution of the form $n^m (p_t n^t + p_{t-1} n^{t-1} + \dots + p_1 n + p_0) s^n$.

What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if **$F(n) = (-2)^n$**

There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java, and C, how many of these 2504 students have not taken a course in any of these three programming languages?

How many elements are in the union of four sets if the sets have 50, 60, 70, and 80 elements, respectively, each pair of the sets has 5 elements in common, each triple of the sets has 1 common element, and no element is in all four sets?