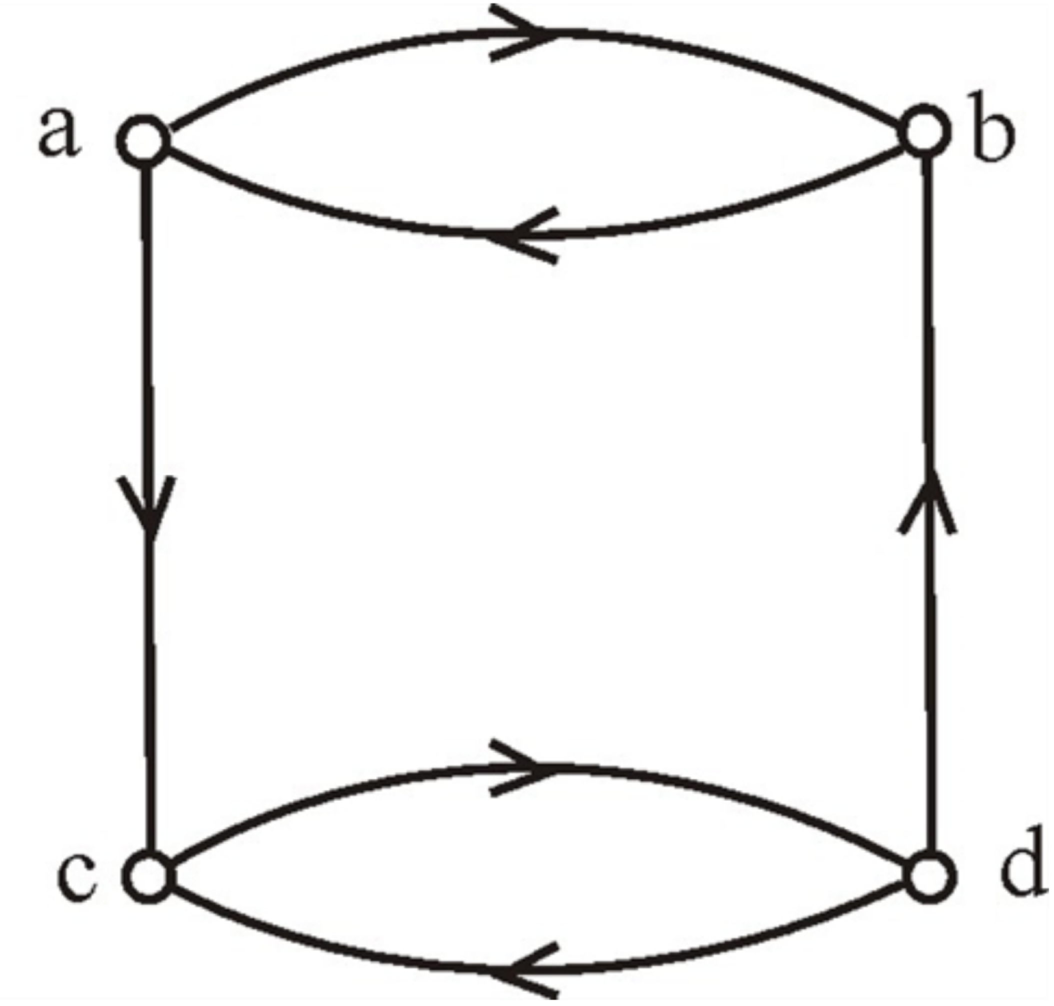
Chapter 9.4

Q3: Let R be the relation {(a, b) | a divides *b*} on the set of integers. What is the symmetric closure of R?

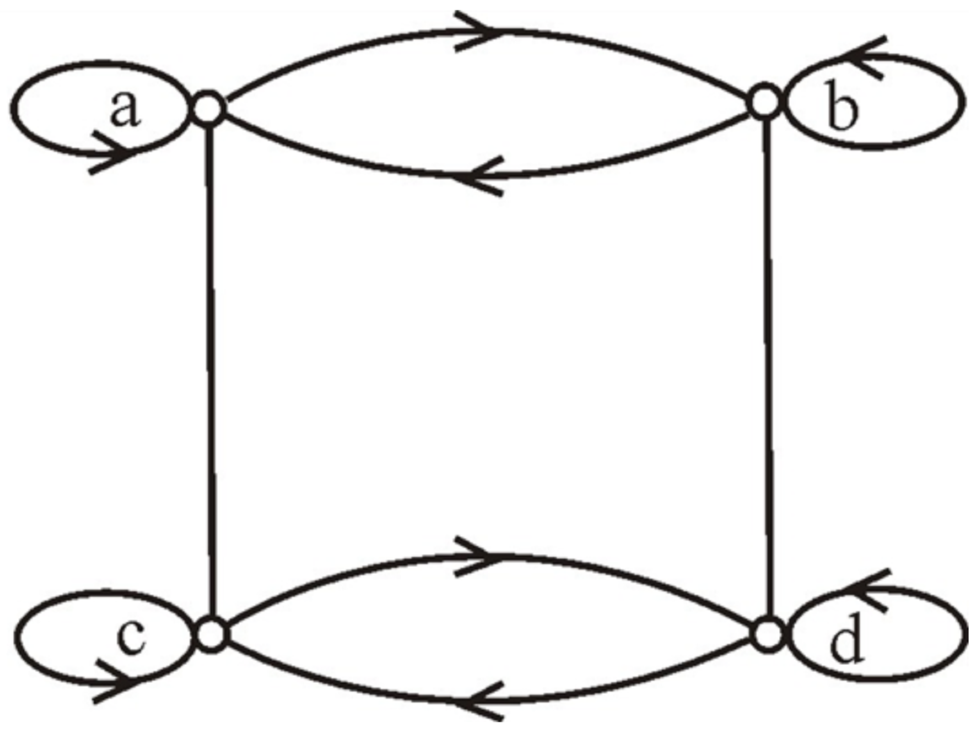
- The symmetric closure of a relation is simply the union of the elements in that relation along with its inverse.
  - Let R= {(a, b) | a divides *b*}
  - Then, R^{-1}= {(a, b) | b divides *a*}
  - The symmetric closure is given by R U R^{-1}
    - {(a, b) | a divides *b*} U {(a, b) | b divides *a*}
- The symmetric closure of the set consists of all the ordered pairs of integers which are composite to each other

Q5: draw the directed graph of the reflexive closure of the relations with the directed graph shown.

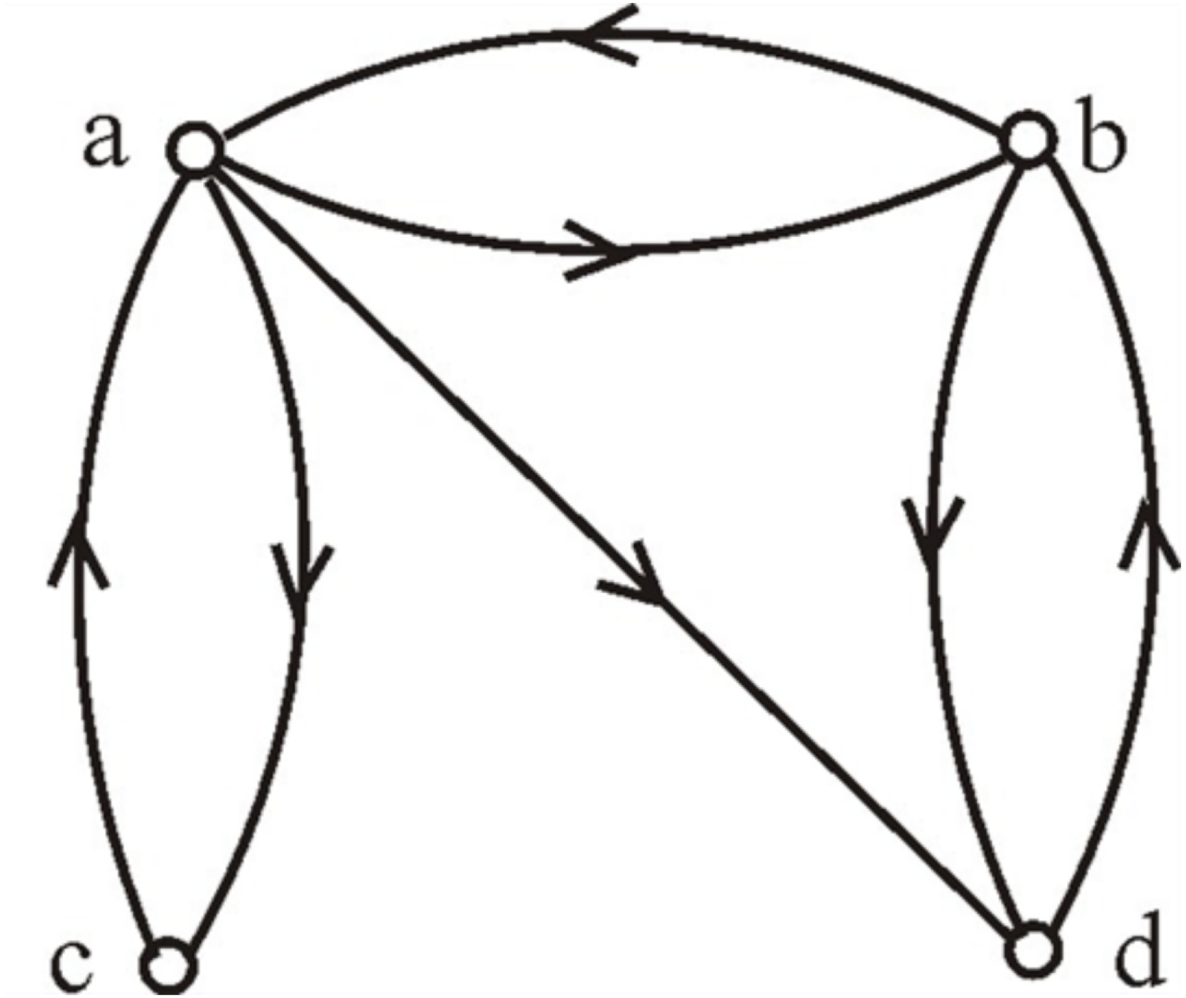


Q5: draw the directed graph of the reflexive closure of the relations with the directed graph shown.

- The directed graph representing the reflexive closure of a relation on a finite set can be constructed by joining a loop at every vertex of the directed graph of the relation.
- The reflexive closure is:

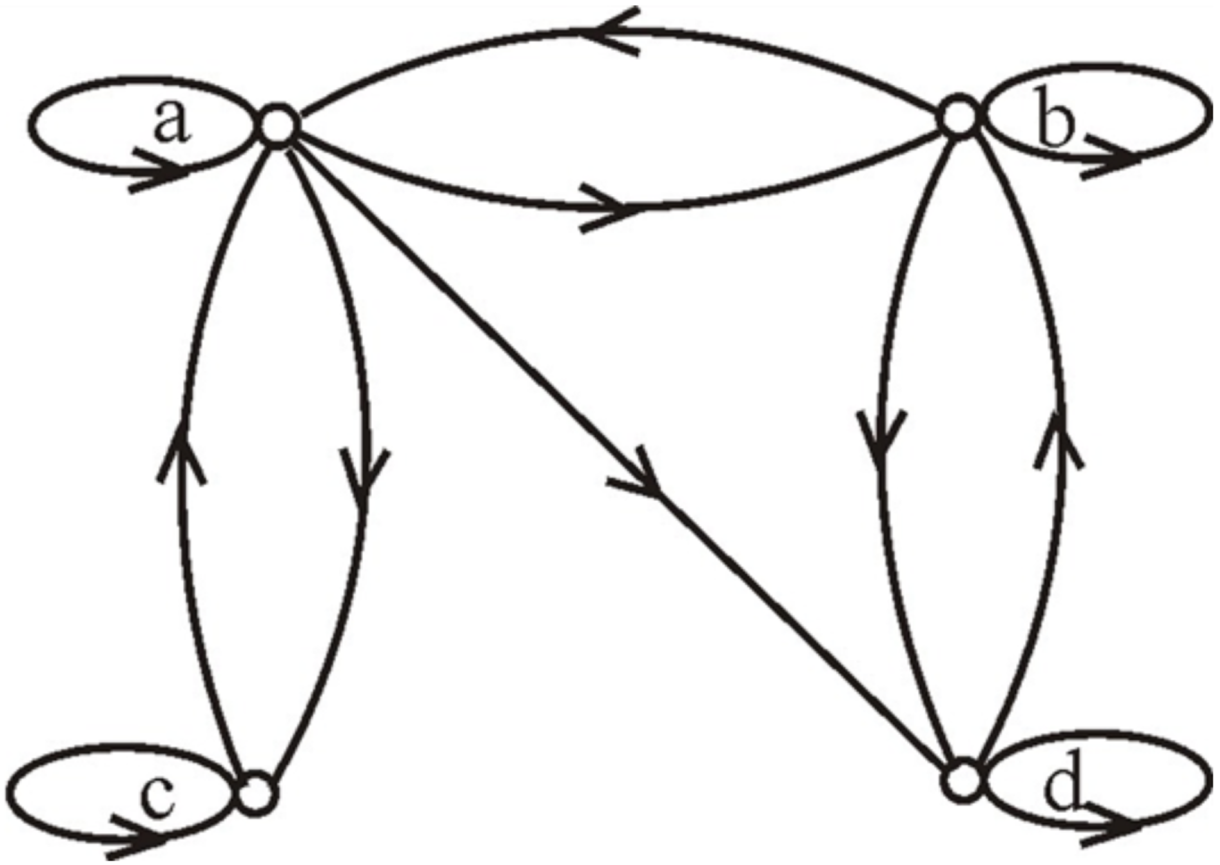


Q7: draw the directed graph of the reflexive closure of the relations with the directed graph shown.

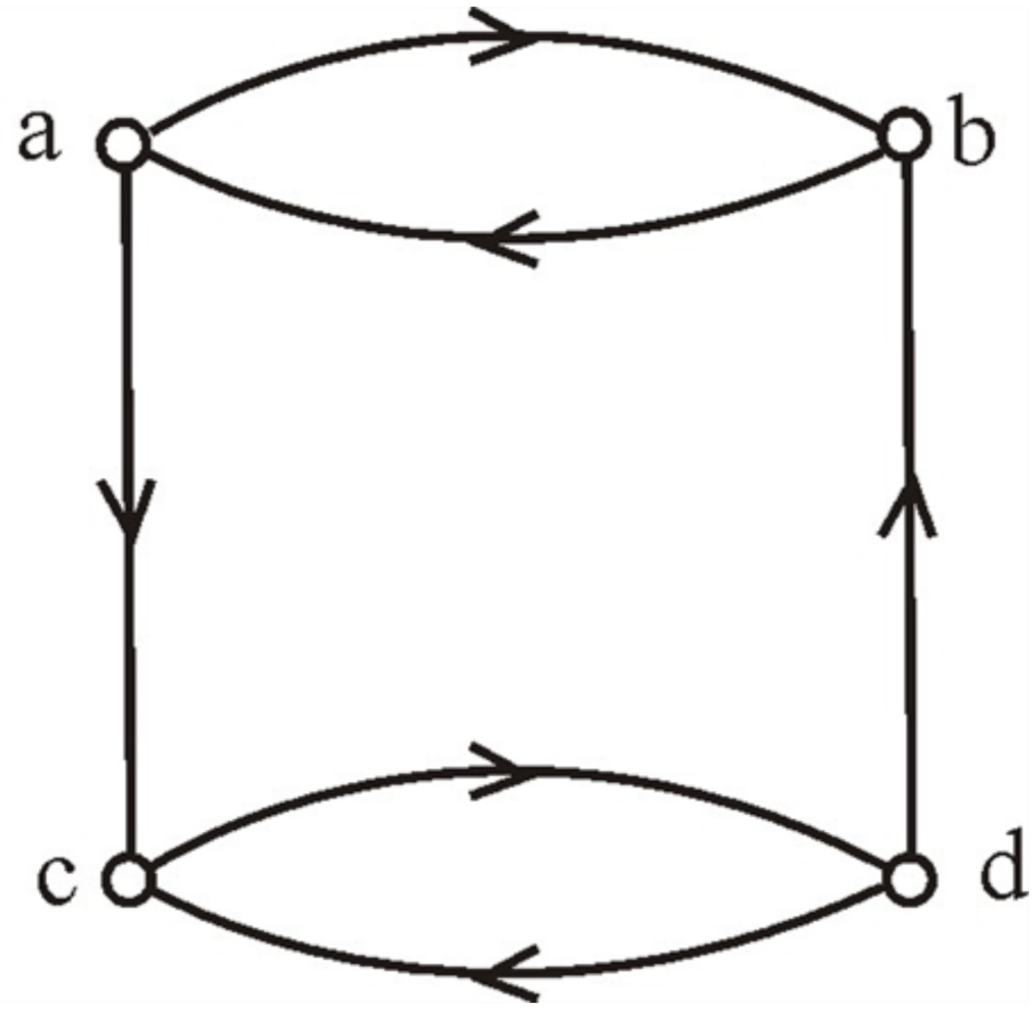


Q7: draw the directed graph of the reflexive closure of the relations with the directed graph shown.

- The reflexive closure is:

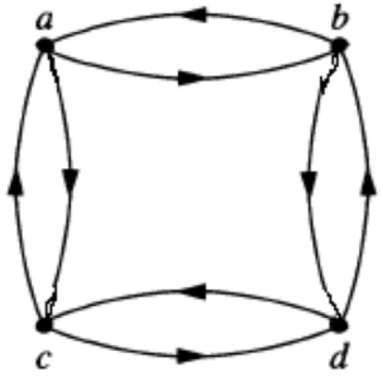


Q9: Find the directed graphs of the symmetric closures of the relations with directed graphs shown



Q9: Find the directed graphs of the symmetric closures of the relations with directed graphs shown

- The symmetric closure of a relation is simply the union of the elements in that relation along with its inverse.
- According to the graph, R = {(a,b), (b,a), (a,c), (c,d), (d,c), (d,b)}
- R is not symmetric
- To make R symmetric, add (c,a) and (b,d) in R
- The directed graph is:



Q13: Suppose that the relation R on the finite set A is represented by the matrix M_R. Show that the matrix that represents the symmetric closure of R is $M_R \vee M^t_R$

(i) If R is a relation defined on a set A, then $R \cup R^{-1}$ is the symmetric closure of R, where

$$R^{-1} = \{(b,a)/(a,b) \in R\}$$

(ii) If R and S are two relations defined on a finite set A and they are represented by the matrices M_R and M_S respectively, then the matrix representing $R \cup S$ is $m_{R \cup S}$ given by

$$m_{R \cup S} = m_R \vee m_S$$

(iii) $m_{R^{-1}} = m_R^T$, transpose of m_R

Q13: Suppose that the relation R on the finite set A is represented by the matrix M_R. Show that the matrix that represents the symmetric closure of R is $M_R \vee M^t_R$

Suppose R is a relation defined on a finite set

Then, $R \cup R^{-1}$ is the symmetric closure of R.

Therefore, the matrix that represents the symmetric closure of R is $m_{R \cup R^{-1}}$

But, $m_{R \cup R^{-1}} = m_R \vee m_{R^{-1}}$ and $m_{R^{-1}} = m_R^T$

Hence, the matrix that represents the symmetric closure of R is $m_R \vee M_R^T$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

A. R^2
B. R^3
C. R^4
D. R^5
E. R^6
F. R^*

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

We know:

(i) A relation R defined on a set $A=\{a_1, a_2, \ldots\ldots, a_n\}$ can be represented by a matrix

$m_R = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, a_j) \in R \\ 0 & \text{if } (a_i, a_j) \notin R \end{cases}$$

(ii) For any positive integer n,

$M_{R^n} = m_R^{[n]} = m_R \square M_R \square \ldots\ldots\ldots\square M_R$ (n times)

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

Let $R=\{(1,3),(2,4),(3,1),(3,5),(4,3),(5,1),(5,2),(5,4)\}$ be a relation on $\{1,2,3,4,5\}$

Then

$$M_R = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

(a) $M_{R^2} = M_R \square M_R$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \square \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Hence $R^2 = \begin{Bmatrix} (1,1),(1,5),(2,3),(3,1),(3,2),(3,3),(3,4), \\ (4,1),(4,5),(5,3),(5,4) \end{Bmatrix}$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

(b) $M_{R^3} = m_{R^2} \square M_R$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \square \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Hence , $R^3 = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(2,1),(2,5),(3,1), \\ (3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(5,1),(5,3),(5,5) \end{Bmatrix}$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

(c) $M_{R^4} = m_{R^3} \square M_R$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \square \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Hence , $R^4 = \begin{Bmatrix} (1,1),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,3),(4,4),(4,5), \\ (5,1),(5,2),(5,3),(5,4),(5,5) \end{Bmatrix}$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

(d) $M_{R^5} = m_{R^4} \square M_R$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \square \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Hence , $R^5 = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,3),(2,4),(2,5), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5), \\ (5,1),(5,2),(5,3),(5,4),(5,5) \end{Bmatrix}$

Q19: Let R be the relation on the set {1, 2, 3, 4, 5} containing the ordered pairs (1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), and (5, 4). Find

(e) $M_{R^6} = m_{R^5} \square M_R$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \square \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Hence , $R^6 = A \times A$, where $A = \{1, 2, 3, 4, 5\}$

(f) $R^* = R \cup R^2 \cup R^3 \cup R^4 \cup R^5$

$= A \times \mathrm{A}$, where $A = \{1, 2, 3, 4, 5\}$

Q25: Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}.

R = {(1, 2), (2, 1), (2, 3), (3, 4), (4, 1)}

If R is a relation defined on a finite set containing n elements, then

$$M_{R^*} = m_R \vee M_R^{[2]} \vee \ldots\ldots\ldots \vee m_R^{[n]}$$

Q25: Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}.

(a) Suppose $R=\{(1,2),(2,1),(2,3),(3,4),(4,1)\}$ is a relation on $\{1,2,3,4\}$. Then

$$M_R=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}$$

$$m_R^{[2]} \quad M_{R^2}=M_R \odot M_R$$

$$=\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\odot\begin{bmatrix}0&1&0&0\\1&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}=\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix}$$

Q25: Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}.

$$M_R^{[3]} = m_R^{[2]} \square\ M_R$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \square \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M_R^{[4]} = m_R^{[3]} \square\ M_R$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \square \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Q25: Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}.

$$M_{R^*} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Hence, $R^* = A \times A$, where $A = \{1, 2, 3, 4\}$