CS275 Discrete Mathematics

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Section 2.2
Set Operations

Assume we have two sets:
A = students who live within 1 mile of school
B = students who walk to classes

Describe the students in each of these sets

- $A \cap B$ All the students in set A and all the students B
- $A \cup B$ Students who live within 1 mile of school and walk to classes
- $A - B$ Students who live within 1 mile of school but **DON'T** walk to classes
- $B - A$ Students who walk to classes but **DON'T** live within 1 mile of school

Prove the identity laws by showing that:

$\mathbf{A} \cup \boldsymbol{U} = \boldsymbol{U}$, $\mathbf{A} \cap \emptyset = \emptyset$

- $A \cup U = U$

  $A \cup U = \{x \mid x \in A \vee x \in U\} = \{x \mid x \in A \vee \boldsymbol{T}\} = \{x \mid \boldsymbol{T}\} = U$

- $A \cap \emptyset = \emptyset$

  $A \cap \emptyset = \{x \mid x \in A \wedge x \in \emptyset\} = \{x \mid x \in A \wedge \boldsymbol{F}\} = \{x \mid \boldsymbol{F}\} = \emptyset$

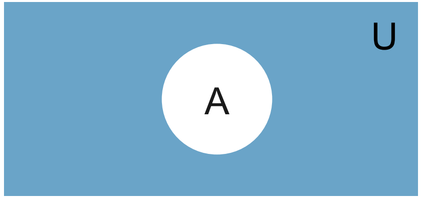
Find the sets A and B if A-B={1,5,7}, B-A={2, 10}, A∩B = {9}

- $A = (A-B) \cup (A \cap B) = \{1,5,7\} \cup \{9\} = \{1,5,7,9\}$

- $B = (B-A) \cup (A \cap B) = \{2, 10\} \cup \{9\} = \{2, 9, 10\}$

Prove the De Morgan laws $\overline{A \cup B} = \overline{A} \cap \overline{B}$ by showing that: 1) each side is a subset of the other side. 2) using a membership table

- $\overline{A}$: The *complement* of set A
  - Let U be the universal set, $\overline{A}$ is U-A



$\overline{A}$

1) $x \in \overline{A \cup B} \equiv x \notin A \cup B \equiv \neg(x \in A \vee x \in B) \equiv \neg(x \in A) \wedge \neg(x \in B) \equiv x \notin A \wedge x \notin B \equiv x \in \overline{A} \wedge x \in \overline{B} \equiv x \in \overline{A} \cap \overline{B}$

Prove the De Morgan laws $\overline{A \cup B} = \bar{A} \cap \bar{B}$ by showing that: 1) each side is a subset of the other side. 2) using a membership table

A	B	$A \cup B$	$\overline{A \cup B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cap \bar{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Let A, B, C be sets. Show that $(\boldsymbol{A}-\boldsymbol{C}) \cap (\boldsymbol{C}-\boldsymbol{B}) = \emptyset$

Suppose $x \in (\boldsymbol{A}-\boldsymbol{C}) \cap (\boldsymbol{C}-\boldsymbol{B})$, then $x \in (\boldsymbol{A}-\boldsymbol{C})$ and $x \in (\boldsymbol{C}-\boldsymbol{B})$

- By definition , if $x \in \boldsymbol{A}-\boldsymbol{C}$, $x \notin C$
- By definition , if $x \in \boldsymbol{C}-\boldsymbol{B}$, $x \in C$

Prove the associative laws by showing that:

$$\mathbf{A} \cup (\mathbf{B} \cup \mathbf{C}) = (\mathbf{A} \cup \mathbf{B}) \cup \mathbf{C}$$

$$x \in A \cup (B \cup C) \equiv (x \in A) \vee (x \in (B \cup C))$$

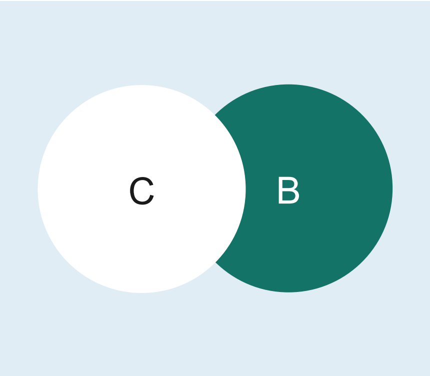
$$\equiv (x \in A) \vee (x \in B \vee x \in C)$$

$$\equiv (x \in A \vee x \in B) \vee (x \in C)$$

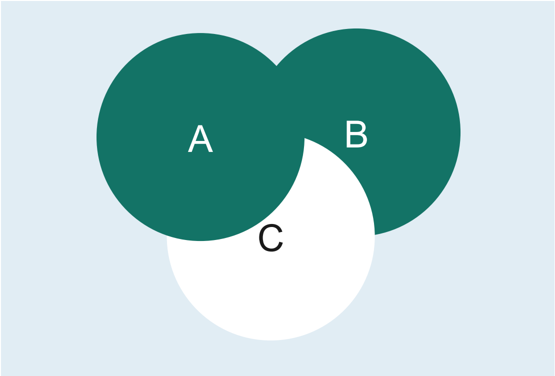
$$\equiv x \in (A \cup B) \cup C$$

Draw the Venn diagrams for the combination of the sets: $\mathbf{A} \cup (\mathbf{B} - \mathbf{C})$

A



B-C

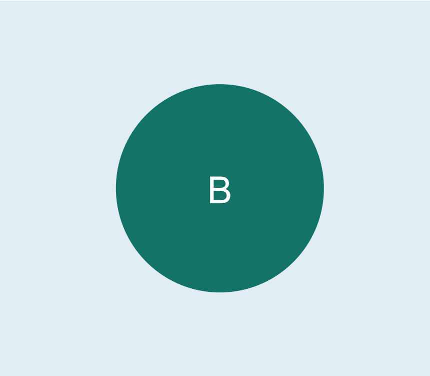


$\mathbf{A} \cup (\mathbf{B} - \mathbf{C})$

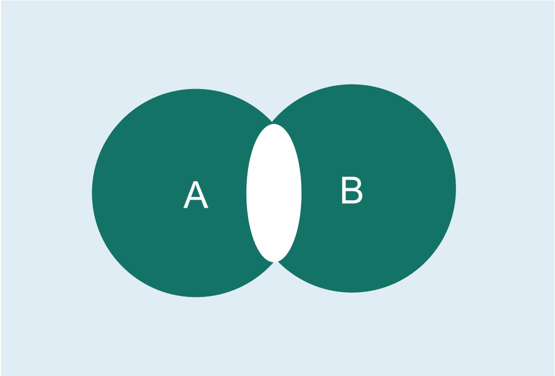
Draw the Venn diagrams for the symmetric difference of the sets A and B

(Symmetric difference of A and B is the set containing those elements in either A or B, but not in both)

A



B



Symmetric Difference

Suppose that the universal set is U={1,2,3,4,5,6,7,8,9,10}. Find the set specified by each of these bit strings.

- 11 1100 1111 {1,2,3,4,7,8,9,10}
- 01 0111 1000 {2,4,5,6,7}
- 10 0000 0001 {1, 10}

Section 2.3
Functions

Why is *f* not a function from *R* to *R* if:

a) f(x) = 1/x, b) f(x) = $\sqrt{x}$, c) f(x) = ± $\sqrt{(x^2+1)}$

- A function f from A to B is a rule which assigns for each element in A, a unique element in B.
- R = the set of all real numbers

- a) f(x) = 1/x — f(0) is not defined
- b) f(x) = $\sqrt{x}$ — For f(x) when x<0 is not defined
- c) f(x) = ± $\sqrt{(x^2+1)}$ — Two distinct values (±) assigned to each x

Find these values:

- $\left\lceil \frac{3}{4} \right\rceil$ 1
- $\left\lfloor \frac{7}{8} \right\rfloor$ 0
- $\left\lfloor \frac{1}{2} \bullet \left[\frac{5}{2} \right] \right\rfloor$ 1

Determine whether the following function *f*:ZxZ->Z is onto

(A function f : A → B is called on-to or surjective if for every element b ∈ B, there is some element a ∈ A with f(a) = b
Z = {⋯−2,−1,0,1,2,3,⋯} = the set of all integers, posi- tive and negative)

- f(m,n) = m+n	onto
- f(m,n) = m^2 + n^2	Not onto
- f(m,n) = m	onto
- f(m,n) = |n|	Not onto
- f(m,n) = m-n	onto

Prove that a strictly decreasing function from *R* to itself is one-to-one

(A function $f(x)$ is said to be strictly decreasing on an interval 1 if $f(b)<f(a)$ for all $b>a$.)

- Prove one-to-one needs to prove $f(a) \neq f(b)$

- Let *f* be a given strictly decreasing function from *R* to itself.
- If $a<b$, then $f(a)>f(b)$
- If $a>b$, then $f(a)<f(b)$
- Thus, if $a \neq b$, then $f(a) \neq f(b)$

Find *f*+*g* and *fg*, where f(x)=x^2 + 1, g(x) = x+2, are functions from *R* to *R*.

- $f+g = x^2+1+x+2 = x^2+x+3$

- $fg = (x^2+1)*(x+2) = x^3+2x^2+x+2$

Let *a* and *b* be real numbers with $a<b$. Use the floor and/or ceiling functions to express the number of integers *n* that satisfy the inequality $a<n<b$.

- We want to use a function to count how many integers are between *a* and *b*
    - Round up *b* and round down *a* to get the upper bound and lower bound of the range
- $\lceil b \rceil - \lfloor a \rfloor - 1$

How many bytes are required to encode *n* bits of data where *n* equals: a) 7, b)17, c)1001, d) 28,800

- Since a byte is eight bits, we are asking to solve ⌈n/8⌉

- a) ⌈7/8⌉ = 1
- b) ⌈17/8⌉ = 3
- c) ⌈1001/8⌉ = 126
- d) ⌈28800/8⌉ = 3600