CS275 Discrete Mathematics

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6.1 The Basis of Counting

The product rule:

Suppose that a procedure can be broken down in to a sequence of two tasks.

- The 1st task can be done in *k* way
- The 2nd task can be done in *m* way

There are $k \cdot m$ ways to do the procedure

Supposed we have 2 shirts and 4 pants, how many outfit combinations we can have?

How about if we also have 3 hats? How many outfit combinations we can have?

A multiple-choice test contains 10 questions. There are four possible answers for each question.

- In how many ways can a student answer the questions on the test if the student answers every question?
- 4^{10}

- In how many ways can a student answer the questions on the test if the student can leave answers blank?
- 5^{10}
  - We have 5 choice for each questions (4 answers + blank)

- How many different three-letter initials can people have?
- 26^3
    - We are asking to fill in the blank of "_ _ _", each blank has 26 possible choices

- How many different three-letter initials are there that <u>begin with an A</u>?
- 26^2
    - We are counting "A _ _"
        - Only need to consider the 2nd and 3rd place.

How many bit strings of **length ten** both **begin and end with a 1**

2^8

- We are asking to fill in the blank of “1 _ _ _ _ _ _ _ _ 1”, each blank has 2 possible choices

The sum rule:

If a task can be done either in one of k ways or in one of m ways, where none of the set of k ways is the same as any of the set of m ways, then there are $k+m$ ways to do the task

How many bit strings with **length not exceeding n**, where n is a positive integer, **consist entirely of 1s**, not counting the empty string?

n bit strings

- n=1   1
- n=2   1, 11
- n=3   1, 11, 111

…

How many strings are there of **lowercase letters of length four or less**, not counting the empty string?

$26^4+26^3+26^2+26$

- length four: 26^4
- length three: 26^3

…

The division rule:

There are n/d ways to do a task if it can be done using a procedure that can be carried out in *n* ways, and for every way *w*, exactly *d* of the *n* ways correspond to way *w*.

If a finite set *A* is the union of pairwise disjoint subsets with *d* elements each, the number of subsets = |A|/d

How many positive integers between 100 and 999 inclusive are divisible by 7?

- There are 900 number in the range
- Every 7 numbers contain one number is divisible by 7
- We can use the division rule by counting “how many pairwise disjoint subsets with *d* elements each”
- $n = \left\lfloor \frac{999-100+1}{7} \right\rfloor = 128$

How many positive integers between 100 and 999 inclusive are odd?

- $n = \left\lfloor \frac{999-100+1}{2} \right\rfloor = 450$

How many positive integers between 100 and 999 inclusive have the same three decimal digits?

- We use product rule here
- We want to fill in the blank of "_ _ _"
- The first place we have 9 choices, 2nd and 3rd only have 1 choice
- $n = 9 \times 1 \times 1 = 9$

How many positive integers between 100 and 999 inclusive are not divisible by 4?

- # of integers divisible by 4, $\mathrm{n}' = \left\lfloor \frac{999-100+1}{4} \right\rfloor = 225$
- # of integers NOT divisible by $\mathrm{n} = (999 - 100 + 1) - n' = 675$

How many positive integers between 100 and 999 inclusive are divisible by 3 or 4?

- Divisible by 3: $n = \left\lfloor \frac{999-100+1}{3} \right\rfloor = 300$
- Divisible by 4: $n = \left\lfloor \frac{999-100+1}{4} \right\rfloor = 225$
- Divisible by 3 and 4: $n = \left\lfloor \frac{999-100+1}{3*4} \right\rfloor = 75$
- Divisible by 3 or 4: $300 + 225 - 75 = 450$

How many positive integers between 100 and 999 inclusive are divisible by 3 but not by 4?

- Divisible by 3: $n = \left\lfloor \frac{999-100+1}{3} \right\rfloor = 300$
- Divisible by 3 and 4: $n = \left\lfloor \frac{999-100+1}{3*4} \right\rfloor = 75$
- Divisible by 3 but not 4 : $300 - 75 = 225$

How many **one-to-one** functions are there from a set with **five elements** to sets with **4** number of elements?

- 0
- A function is one-to-one, if each element only has a **unique** image
- We have 5 elements in the domain and 4 elements in the range (image). Two elements will need to have the same image.

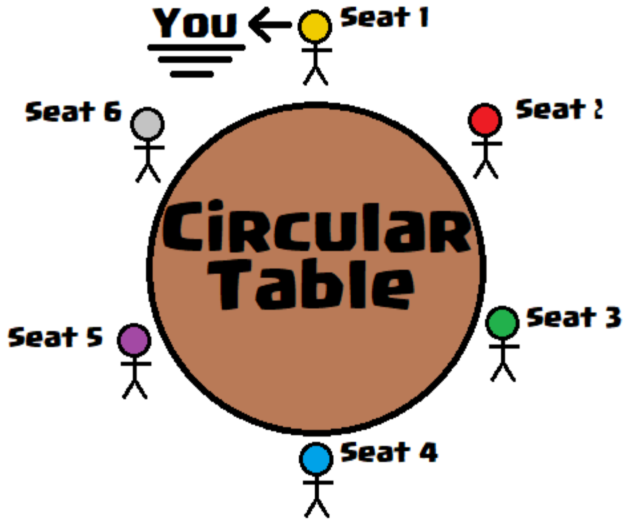
How many **one-to-one** functions are there from a set with **five elements** to sets with **5** number of elements?

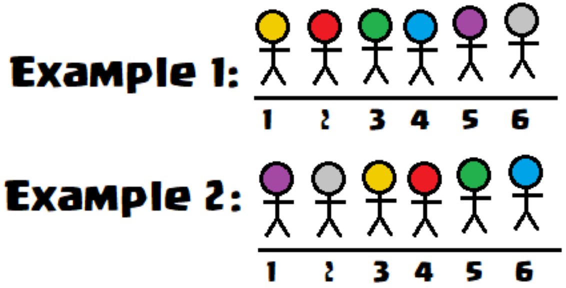
- The 1^{st} element in the domain have 5 possible ways
- The 2^{nd} element in the domain have 4 possible ways
- The 3^{rd} element in the domain have 3 possible ways
- …
- $n = 5 \times 4 \times 3 \times 2 \times 1 = 120$

How many **one-to-one** functions are there from a set with **five elements** to sets with **6** number of elements?

- The 1st element in the domain have 6 possible ways
- The 2nd element in the domain have 5 possible ways
- The 3rd element in the domain have 4 possible ways
- …
- $n = 6\times5\times4\times3\times2 = 720$

How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?





How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

- Since it doesn't matter what seat you sit in, the only thing matters is the order of people
- We can reformat the question to fill in the blank of "_ _ _ _ _ _"
- The 1st place is you, the the 2nd place has 5 options, the 3rd has 4, …
- We have $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways to sit

How many ways are there to seat six people around a circular table where two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

- Since the question also states “two seatings are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors”
- We have $(5\times4\times3\times2\times1)/2 = 60$ ways to sit