

### **6.2 Pigeonhole Principle**

**A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.**

A ) How many socks must he take out to be sure that he has at least two socks of the same color?

B) How many socks must he take out to be sure that he has at least two black socks?

**Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.**

**Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.**

**How many numbers must be selected from the set  $\{1, 2, 3, 4, 5, 6\}$  to guarantee that at least one pair of these numbers add up to 7?**

**Suppose that every student in a discrete math class of 25 students is a freshman, a sophomore, or a junior. Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.**

**Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.**

### **6.3 Permutation and Combination**

**Find the value of each of these quantities.**

A)  $P(8,1)$

B)  $P(8,8)$

C)  $P(10,9)$

**Find the value of each of these quantities.**

A)  $C(8,4)$

B)  $C(8,0)$

A group contains  $n$  men and  $n$  women. How many ways are there to arrange these people in a row if the men and women alternate?

In how many ways can a **set of five letters** be selected from the English alphabet?

**How many subsets with more than two elements does a set with 100 elements have?**

(Recall: if a set  $S$  has  $n$  elements, the power set of  $S$  has  $2^n$  elements)

**A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes**

A) are there in total?

B) contain exactly two heads?

C) contain at most three tails?

D) contain the same number of heads and tails

**How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?** [*Hint: First position the men and then consider possible positions for the women.*]

**The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain**

a) exactly one vowel?

b) exactly two vowels?

c) at least one vowel?

d) at least two vowel?